CBCS SCHEME

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15MAT31

Third Semester B.E. Degree Examination, Jan./Feb. 2023
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the Fourier series of

$$f(x)=\begin{cases}1+\dfrac{4x}{3} & \text{in } -\dfrac{3}{2}<x\le 0\\[2mm] 1-\dfrac{4x}{3} & \text{in } 0\le x\le \dfrac{3}{2}\end{cases}$$

Hence deduce that $\dfrac{\pi^2}{8}=\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\ldots$ **(08 Marks)**

b. Obtain the constant term and the first two coefficients in the Fourier cosine series for y using the following table:

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(08 Marks)

OR

2 a. Obtain the Fourier series for

$$f(x)=\begin{cases}-K & \text{in } (-\pi,0)\\ +K & \text{in } (0,\pi)\end{cases}.$$

Hence deduce that $\dfrac{\pi}{4}=1-\dfrac{1}{3}+\dfrac{1}{5}-\dfrac{1}{7}+\ldots..$ **(06 Marks)**

b. Find a half range Fourier Cosine Series for $f(x)=(x-1)^2$ in $0\le x\le 1$. **(05 Marks)**

c. Given the following table:

x°	0	60°	120°	180°	240°	300°
y	7.9	7.2	3.6	0.5	0.9	6.8

Obtain the Fourier series neglecting the terms higher than first harmonics. **(05 Marks)**

Module-2

3 a. Find the Fourier transform of the function

$$f(x)=\begin{cases}1 & \text{for } |x|\le a\\ 0 & \text{for } |x|>a\end{cases}$$

where 'a' is a positive real constant. Hence evaluate $\int_0^\infty \dfrac{\sin ax}{x}dx$. **(06 Marks)**

b. Find the Z-T of $\cosh n\theta$. **(05 Marks)**

c. Solve by using z-transforms $u_{n+2}+2u_{n+1}+u_n=n$ with $u_0=0=u_1$. **(05 Marks)**

OR

4 a. Find the Fourier sine and cosine transform of $f(x) = e^{-ax}, a > 0, x \neq 0$. **(06 Marks)**

b. Find the Fourier transform of the function

$$f(x) = \begin{cases} 1-|x| & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$ **(05 Marks)**

c. Given $\pi(z) = \dfrac{4z^2 - 2z}{(z-1)(z-2)^2}$. Find u_n. **(05 Marks)**

Module-3

5 a. Calculate the coefficient of correlation for the following data:

x (Height of Father in inches)	65	66	67	68	69	70	72
y (Height of their Son in inches)	67	68	65	68	72	69	71

(06 Marks)

b. Fit an exponential curve of the form $y = ae^{bx}$ by the method of least squares for the following data:

Number of petals	5	6	7	8	9	10
Number of flowers	133	55	23	7	2	2

(05 Marks)

c. Show that a root of equation $x^3 + 5x - 11 = 0$ lies between 1 and 2. Find the root by Newton-Raphson method. (Carry out 3 iterations) **(05 Marks)**

OR

6 a. Obtain regression line of y on x for the following data:

x	36	23	27	28	28	29	30	31	33	35
y	29	18	20	22	27	21	29	27	29	28

(06 Marks)

b. Fit a parabola $y = a + bx + cx^2$ for the data:

x	0	1	2	3	4
y	1	1.8	1.3	2.5	2.3

(05 Marks)

c. Compute real root of $x \log_{10} x - 1.2 = 0$ between 2 and 3 using Regula-Falsi method. Carry out three iterations. **(05 Marks)**

Module-4

7 a. Using suitable interpolation formula, find y(38) and y(85) for the following data:

x	40	50	60	70	80	90
y	184	204	226	250	276	304

(06 Marks)

b. Construct an interpolating formula for the data given below using Newton's divided difference interpolation formula.

x	2	4	5	6	8	10
f(x)	10	96	196	350	868	1746

(05 Marks)

c. By dividing the range into 6 equal parts, find the approximate value of $\int_0^{\pi} e^{\sin x} dx$ using Simpson's $\frac{1}{3}^{rd}$ rule. **(05 Marks)**

OR

8 a. Given Sin 45° = 0.7071, Sin 50° = 0.7660, Sin 55° = 0.8192, Sin 60° = 0.8660. Find Sin 57° using an appropriate interpolation formula. **(06 Marks)**

b. Use Lagrange's interpolation formula to find f(9), given the data:

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

(05 Marks)

c. Evaluate $\int_0^1 \frac{1}{1+x}\,dx$ taking seven ordinates by applying Simpson's $\frac{3}{8}^{th}$ rule. **(05 Marks)**

Module-5

9 a. If $\vec{f} = (2y+3)\hat{i} + xz\hat{j} + (yz - x)\hat{k}$, evaluate the integral $\int_C \vec{f}\cdot d\vec{r}$ where 'c' is the curve $x = 2t^2$, $y = t$, $z = t^3$ from the point (0, 0, 0) to the point (2, 1, 1). **(06 Marks)**

b. Using the divergence theorem, evaluate $\int_S \vec{f}\cdot\hat{n}\,ds$ where $\vec{f} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and 's' is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. **(05 Marks)**

c. Find the extremals of the functional $\int_{x_0}^{x_1} \frac{(y')^2}{x^3}\,dx$. **(05 Marks)**

OR

10 a. Evaluate $\int_C xydx + xy^2dy$ by Stoke's theorem where 'c' is the square in the xy - plane with vertices (1, 0), (−1, 0), (0, 1), (0, −1). **(06 Marks)**

b. Verify the Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$ where 'c' is the closed curve of the region bounded by $y = x$ and $y = x^2$. **(05 Marks)**

c. Solve the variational problem $\delta\int_1^2 [x^2(y')^2 + 2y(x+y)]dx = 0$ given $y(1) = y(2) = 0$. **(05 Marks)**

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